Reasoning with Many-Valued, Spatial and Temporal Logics with SOLE

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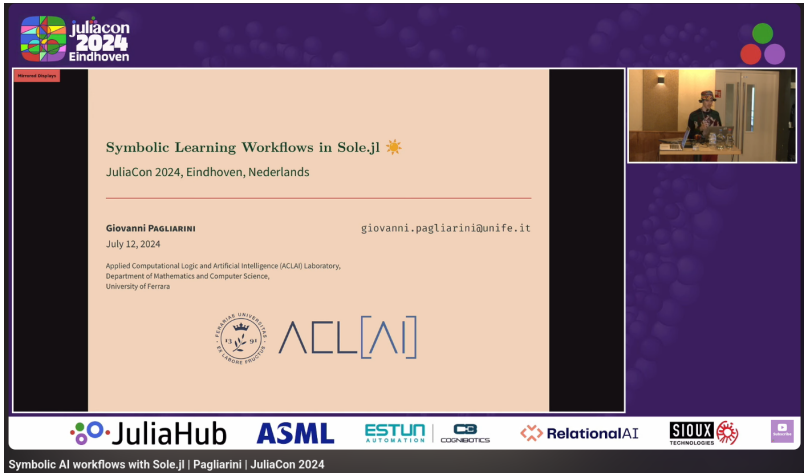
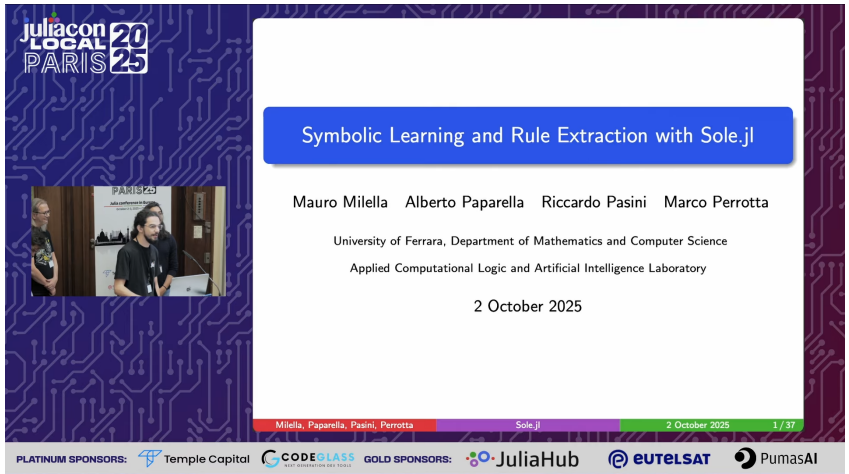
SOLE at JuliaCon

(a) JuliaCon 2023



(b) JuliaCon 2024



(c) JuliaCon Local 2025

Today's talk

Other conferences:

- Motivation
- Applications
- Mathematical framework
- ...
- **Implementation in Julia (:**

Today's talk:

- New stuff in SOLE

Questions on left column?

Reach out anytime! (:

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Today's talk

SoleLogics.jl

https://github.com/aclai-lab/SoleLogics.jl

SoleReasoners.jl

https://github.com/aclai-lab/SoleReasoners.jl

using SoleLogics.jl: ManyValuedLogics

A broad family of Many-Valued Logics

A many-valued logic can be interpreted through its corresponding algebra.

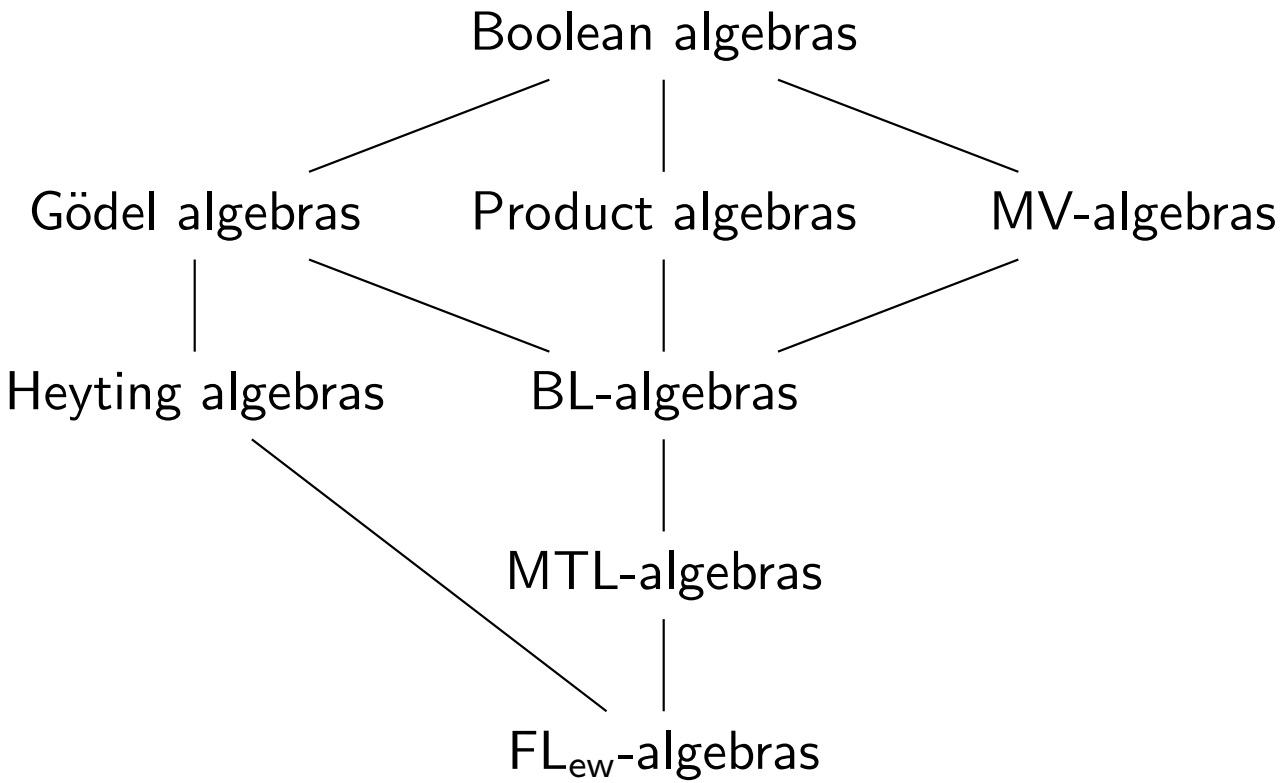

Figure: A partial taxonomy of well-known many-valued algebras.

The sweet spot: $\mathrm{FL_{ew}}$-algebras

Definition

$\mathrm{FL_{ew}}$-algebras

$$\mathcal{A} = \langle A, \cap, \cup, \cdot, 0, 1 \rangle$$

are defined over **bounded commutative residuated lattices**, where:

- $\langle A, \cap, \cup, 0, 1 \rangle$ represents a **bounded complete lattice**
- $\langle A, \cdot, 1 \rangle$ is a **commutative monoid**
- We can define an **implication** $\hookrightarrow$ (the residuation property holds)

$$\alpha \hookrightarrow \beta = \sup\{\gamma \mid \alpha \cdot \gamma \preceq \beta\}$$

$\mathcal{A}$ is a **chain** if $\langle A, \preceq \rangle$ is a total order, **finite** if A is finite.

The sweet spot: $\mathrm{FL}_{\mathrm{ew}}$-algebras

Take-home message:

- Lattice structure $=$ **(partial) order**
- Monoid operator $=$ **conjunction** $(\wedge)$
- Implication is always **derived** from conjunction

Some examples of lattice structures

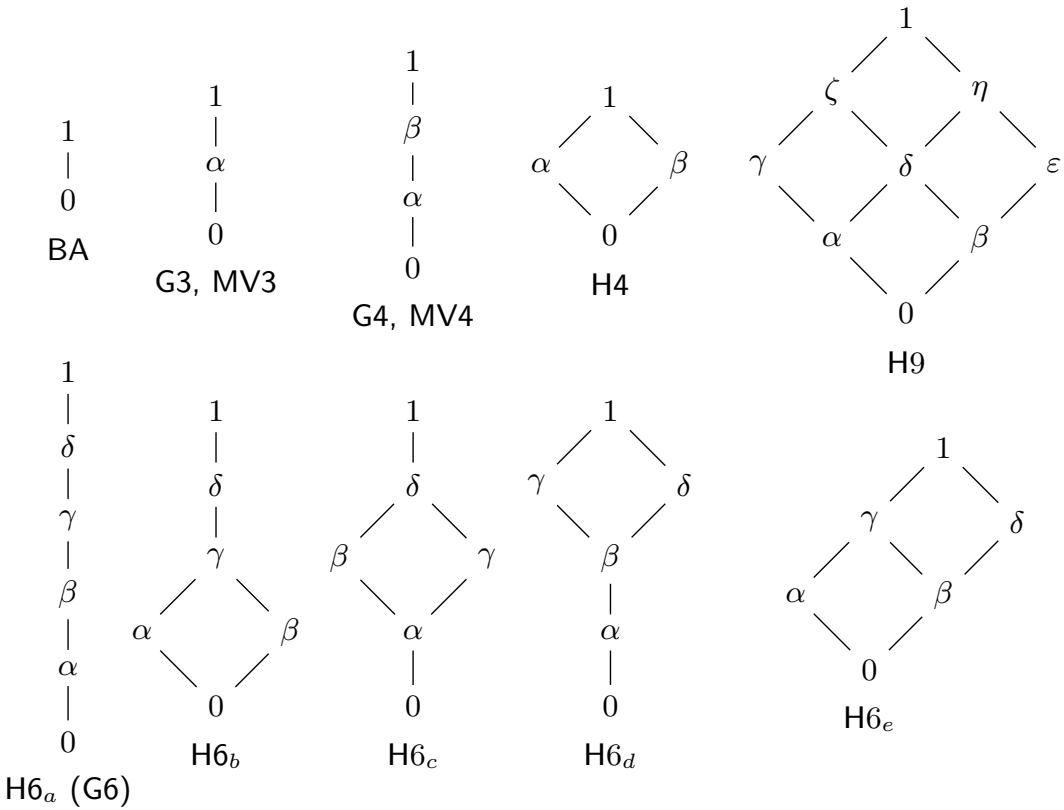

Figure: Examples of (finite) lattice structures (i.e., partial orders).

Some examples of monoid operators

We can use the monoid operator to model continuous t-norms:

- Łukasiewicz t-norm:

$$x * y = max(0, x + y - 1)$$

- Gödel t-norm:

$$x * y = min(x, y)$$

- Product t-norm:

$$x * y = x \cdot y \quad \text{(product of reals)}$$

Some examples of (derived) implications

For $x \leq y, x \Rightarrow y = 1$.

For $x > y$,

- Łukasiewicz implication:

$$x \Rightarrow y = \ y - x$$

- Gödel implication:

$$x \Rightarrow y = y$$

- Goguen implication:

$$x \Rightarrow y = y/x \quad \text{(residuum of product conjunction)}$$

using SoleReasoners.jl

Many-Valued Multi-Modal Logic

Definition

Let $\mathcal{A}P$ be a set of propositional letters, $\mathcal{A}$ a complete $\mathrm{FL}_{\mathrm{ew}}$-algebra. Well-formed formulas of *Multi-Modal Logic* $\mathrm{FL}_{\mathrm{ew}}$-$K_n$ are obtained as:

$$\varphi ::= \alpha \mid p \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \to \psi \mid \langle X_i \rangle \varphi \mid [X_i]\varphi,$$

for $1 \leq i \leq n$, $p \in \mathcal{A}P$, and $\alpha \in \mathcal{A}$.

- We need a function $\widetilde{V}$ to evaluate each operator (**semantics**)
- $\langle X_i \rangle$ reads "it can be the case", $[X_i]$ reads "it must be the case"
- Useful for treating temporal and spatial logics!

The α-satisfiability problem

Definition

A formula φ of a many-valued logic is α-*satisfied* by some model $\widetilde{M}$ iff:

$$\widetilde{V}(\varphi) \succeq \alpha.$$

Definition

A formula φ is α-*satisfiable* iff there exists a model in which it is α-satisfied; resp., a formula is α-*valid* if it is α-satisfied in every model.

To check (α)-satisfiability or validity, we use an **analytic tableau**.

Example: $\Diamond(p \wedge \alpha) \wedge \alpha$ is α-satisfiable in G3

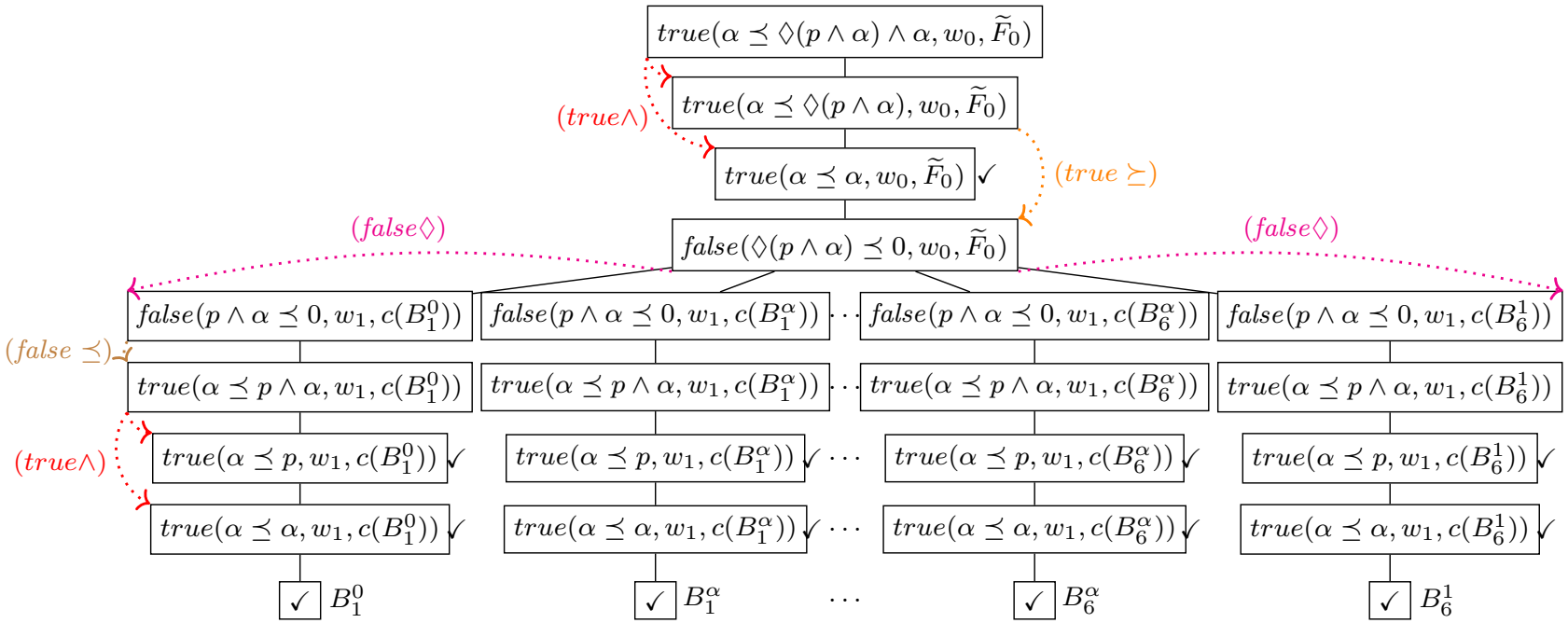

Figure: We only needed to fully expand one branch to prove α-satisfiability; here, we choose to show all possible branches for example purposes.

Example: $\Diamond(p \wedge \alpha) \wedge \alpha$ is not α-satisfiable in MV3

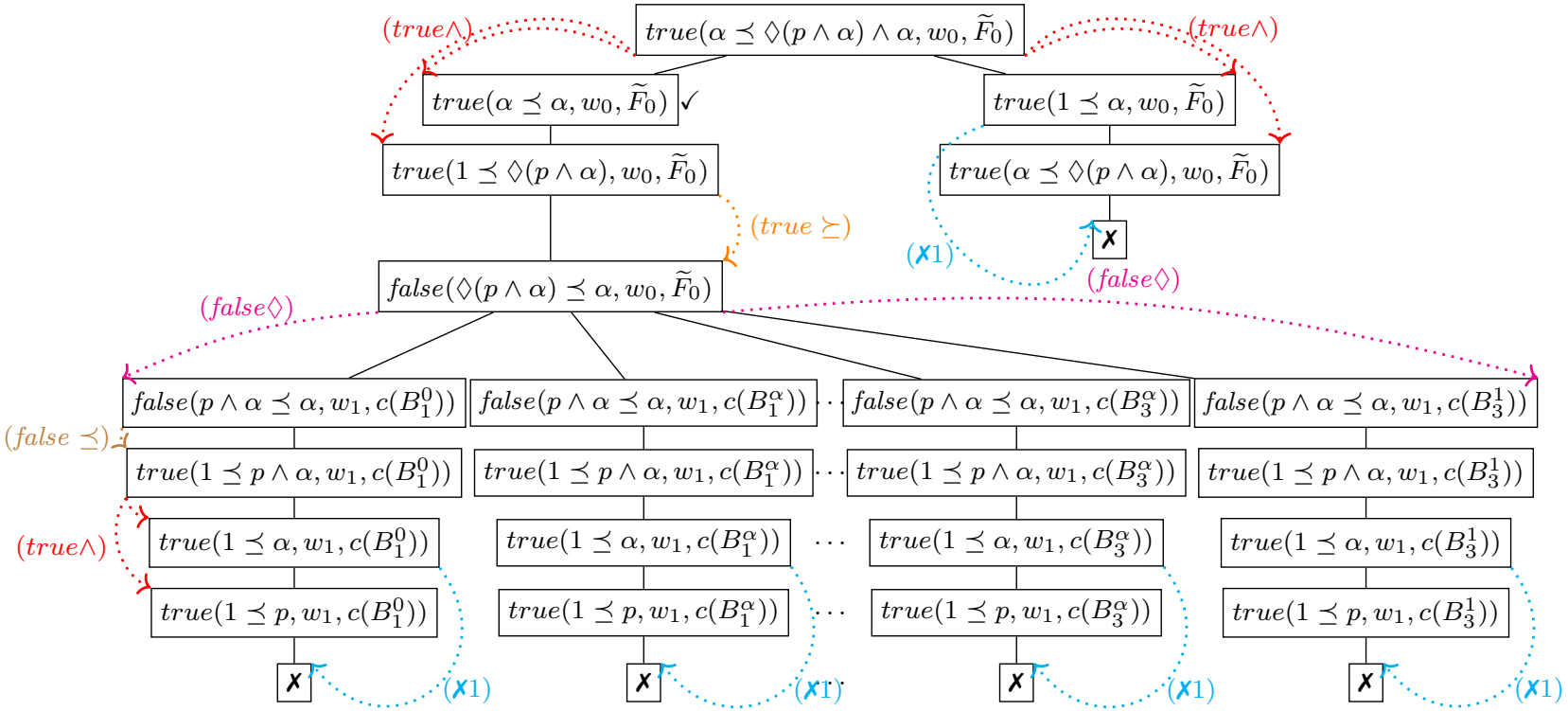

Figure: Applying the rules of the tableau system results in closing all branches.

Reasoning with SOLE

Live demo

Reasoning with SOLE

```
julia> using SoleLogics

julia> using SoleLogics.ManyValuedLogics

julia> using SoleReasoners

julia> using SoleLogics: LTLFP_F

julia> using SoleLogics.ManyValuedLogics: G3, Ł3, α

julia> p = Atom("p")
Atom{String}: p

julia> diamondF = diamond(LTLFP_F)
DiamondRelationalConnective{SoleLogics._GreaterRel}: ⟨>⟩

julia> φ = ∧(diamondF(∧(p,α)),α)
SyntaxBranch: ⟨>⟩(p ∧ α) ∧ α

julia> alphasat(MVLTLFPTableau, α, φ, G3)
true

julia> alphasat(MVLTLFPTableau, α, φ, Ł3)
false
```

Figure: How to code the previous example using SOLE.

Limitations and future work

Conclusions

Limitations

- The reasoning tool only supports **finite** many-valued logics
- Most real-work applications only need (very simple) fuzzy logics

Future work

- Reasoning support for **continuous**-chains (i.e., **BL-chains**)
- **Ad hoc** reasoning tools for simpler logics
- Advanced techniques for fuzzy and many-valued **model checking**

Thank you for your attention!

Questions?